Geometry Transformations

Translations and Rotations

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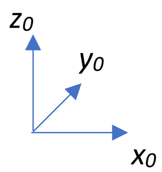
This document describes the use of transformations in robot kinematics as it relates to the MT-dh-serial-kinematics library.

Homogenous transformations applied to orthonormal co-ordinate frames

Start with a coordinate frame at a given origin, oriented as shown (Frame 0):

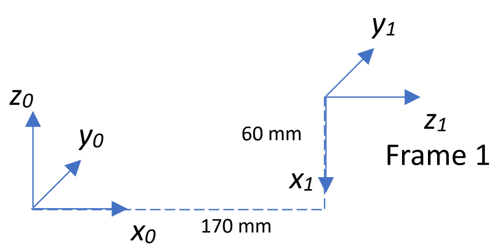


Say we want a new frame relative to the Frame 0, that is translated by 170 mm in the x-direction and 60mm in the z-direction, and rotated by π/2 radians (90 °) about the y-axis so that the x-axis faces downward (using the right-hand rule); the transformation matrix that describes the position and orientation of this this new frame (Frame 1) relative to Frame 0 is given by:

T_0^1 = *transl(…, 170, 0, 0) * transl(…, 0, 0, 60) * troty(…, π/2)*

OR

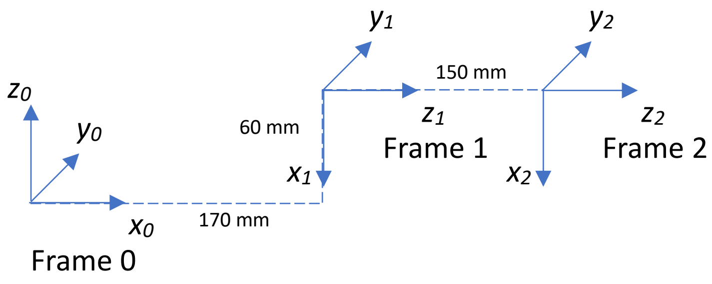
T_0^1 = *transl(…, 170, 0, 60) * troty(…, π/2)*



Note that the order of transformations is important, e.g., troty(…, π/2) * transl(…, 170, 0, 60) will produce a new frame 170 mm below Frame 0, and 60mm in front, with the x-axis facing downwards. This is because each successive transformation creates a new intermediate frame, from which the next transformation is obtained in relation.

Let's add a third frame (Frame 2) relative to Frame 1. To keep it simple we will only translate this frame by 150 mm in the z-direction (of Frame 1). The transformation matrix is given by:

T_1^2 = *transl(..., 0, 0, 150)*



The transformation matrix that describes the position and orientation of Frame 2 relative to Frame 0 is given by:

$T_0^2 = T_0^1 * T_1^2$

This is the basis of serial chain kinematics. The Denavit-Hartenberg (D-H) method wraps the process in an algorithm that standardises the assignment of coordinate frames to robot/manipulator joints in a systematic and consistent manner, and the MT-dh-serial-kinematics library provides methods to facilitate the creation of robots/manipulators using this method. The provided geometrical transformation functions can also be used in conjunction.